AN ALTERNATIVE APPROACH FOR THE ASSETS PRICING IN UNSTABLE FINANCIAL MARKETS

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Abstract:
An alternative approach to stochastic calculus for a financial model on some imperfect and unstable financial markets is proposed. Following the most recent instrument for the financial modeling, we study the solvability of a class of forward-backward stochastic differential equations (FBSDE) in the framework of McShane stochastic calculus, in some general hypothesis on the initial value and the coefficient functions.

Key words: *stochastic finances, stochastic modeling, forward-backward stochastic differential equations, option pricing, contingent claims valuation, zero-bond pricing.*

JED Classification: C02 (Primary), G13 (Secondary)

Introduction

Finance is one of the fastest developing areas in the modern banking and corporate word. This, together with the sophistication of modern financial products, provides a rapidly growing impetus for new mathematical models and modern mathematical methods.

When an evolution of a financial asset is affected by exterior disturbances, its time-development can often be described by a system of ordinary differential equations, provided that the disturbances are smooth functions. But, for round reasons financial analysts want to apply the theory when the noises belong to a larger class, including for example white noise. A unified theory was give by E.J.McShane ([18],[19]) who introduced so called belated integrals and stochastic differential systems which enjoying the following three properties: inclusiveness, consistency and stability. McShane's calculus had proved to very valuable in modeling and it finding applications in physics, engineering and economics.

In the last years, more sophisticated models are available on some restrictive financial hypothesis, but this hypothesis are not satisfied on some transition financial markets as in East Europe, where are more unpublished information, over quoted initial values and government financial interventions. Moreover, the evolution on these markets is characterized by some "smoothed" life-time and some very "noises" life-time and this time periods are hard unexpected. For these reasons, we propose an approach somehow, more general as there for a free financial market.

The classic stochastic approach for the financial models has used the framework developed by Ito to deal with the resulting stochastic differential equations (SDE), based on the idea that a Wiener stochastic process is used for the external disturbances. Then, more authors supposed an semimartingale process for the external noises which make very complicated stochastic calculus. On the other side, E.J.McShane developed a more simple integration calculus using the Ito-belated integrals. In 1979, Ph.Protter showed that the McShane calculus is equivalent with the integration with respect to a semimartingale process. Somehow, this situation is similar with the fact that a Riemann-Stieltjes integral can be considered as a Lebesque integral in some adequate framework, but practically we prefer to use the Riemann integration calculus as to be moresimple. In [..] was given, in details, mathematical properties for this frame and we can sustain that the McShane's Calculus had proved to be very valuable in modeling and in finding application in finances under canonical form.

Preliminaries

In first year of 70's, E.J.McShane introduced so called belated integrals and stochastic differentials and differential systems which enjoying the following three properties: inclusiveness, consistency and stability. McShane's calculus had proved to very valuable in modeling and is finding applications in physics, engineering and economics.

A stochastic integral equations by McShane type is one of the following form:

$$X(t)=x(0)+\int_0^t f(s,X(s))ds+\sum_{j=1}^{r}\int_0^t g_j(s,X(s))dz_j(s)+ \\ +\sum_{j,k=1}^{r}\int_0^t h_{jk}(s,X(s))dz_j(s)dz_k(s) \qquad (1)$$

where the integrals are belated or McShane integrals.

On the above equation, we recall some specifically results of the McShane stochastic calculus. Let (Ω,F,P) be a complete probability space and let $\{F_t, 0\le t\le a\}$ be a family of complete $\sigma-$subalgebras of F such that $0\le s\le t\le a$ then $F_s\subseteq F_t$. Every process denoted by z with diferent affixes will be a real valued second order stochastic process adapted to $\{F_t, 0\le t\le a\}$ (i.e. $z(t)$ is F_t-measurable for every $t\in[0,a]$) and

$$|E[(z(t)-z(s))^m/F_s]|\le K(t-s)$$

a.s., whenever $0\le s\le t\le a, m=1,2,4,$ for a positive constant K having a.s. continuous sample functions (and we say that the process satisfies a K-condition).

It is known (see [18]) that if $f:[0,a]\to L_2$ is a measurable process adapted to the F_t and if $t\to\|f(t)\|$ is Lebesgue integrable on $[0,a]$, then if z_1 and z_2 satisfy a K-condition, the McShane integrals $\int_0^a f(t)dz_1(t)$ and $\int_0^a f(t)dz_1(t)dz_2(t)$ exist and the following estimates are true

$$\|\int_0^a f(t)dz_1(t)\|\le C\{\int_0^a\|f(t)\|^2dt\}^{\frac{1}{2}} \qquad (2)$$

$$\|\int_0^a f(t)dz_1(t)dz_2(t)\|\le C\{\int_0^a\|f(t)\|^2dt\}^{\frac{1}{2}} \qquad (3)$$

where $C=2Ka^{\frac{1}{2}}+K^{\frac{1}{2}}$.

An important class of McShane stochastic differential equations is the class of equation which have a canonical extension or a canonical form (as in McShane a), i.e. the equation (1) with the special case when

$$h_{jk}(t,X(t))=\frac{1}{2}\sum_{i=1}^{n}\frac{\partial g_j(t,X(t))}{\partial X^i}\cdot g_k(t,X(t)),\ X=(X^1,\dots,X^n).\ (4)$$

Among to this forward equations, in the optimal stochastic control appear some backward differential equations as the following

$$\begin{aligned}&Y(t)+\int_t^1 f(s,Y(s),Z(s))ds+\sum_{j=1}^{r}\int_t^1[g_j(s,Y(s),Z(s))]dz_j(s,\omega)+\\&+\sum_{j,k=1}^{r}\int_t^1[h_{jk}(s,Y(s),Z(s))]dz_j(s)dz_k(s)=Y_1\end{aligned}\qquad(5)$$

where $\{z_j(t),0\le t\le 1\}$, $j=1,2,\dots,r$ is a stochastic process defined on the probability space (Ω,F,P) with the natural filtration $\{F_t,0\le t\le 1\}$ and Y_1 is a given F_1-measurable random variable such that $E|Y_1|^2<\infty$. Moreover, f is a mapping from $\Omega\times[0,1]\times R\times R$ to R which is assumed to be $P\otimes B\otimes B\setminus B$-measurable, where P is the σ-algebra of F_t-progressively measurable subsets of $\Omega\times[0,1]$. Also g is a mapping from $\Omega\times[0,1]\times R$ to R which is assumed to be $P\times B\setminus B$-measurable.

We remark that in the case of backward stochastic differential equations by the McShane type we have a canonical extension when replace the functions h_{jk} as above.

In this context we consider the following forward-backward stochastic differential equation by the McShane type

$$\begin{cases}X_t=X_0+\sum_{j=1}^{r}\int_0^t a_j(s,X(s),Y(s),Z(s))dz_j(s)+\\ \quad+\sum_{j,k=1}^{r}\int_0^t b_{jk}(s,X(s),Y(s),Z(s))dz_j(s)dz_k(s)\\ Y_t=\sum_{j=1}^{r}\int_t^1 f_j(s,X(s),Y(s),Z(s))dz_j(s)+\\ \quad+\sum_{j,k=1}^{r}\int_t^1 g_{j,k}(s,X(s),Y(s))dz_j(s)dz_k(s)-h(Y_1)\end{cases}\qquad(6)$$

with $a,b,f,g:\Omega\times(0,1)\times R\times R\times R\to R$, $h:R^+\to R$ and the following hypotheses (which extend the result of Athanassov 1990 [2] for ordinary differential equations and includes other results on FBSDE):

i) a,b,f and g is $P\otimes B\otimes B\otimes B$ measurable functions;

ii) $\varphi(\cdot,0,0,0)\in M^2((0,1),R)$, where φ is any functions a,b,f or g ($M^2(0,1)$ is the set of all stochastic process which are square McShane integrable on $[0,1]$ and F_t-measurable for $0\le t\le 1$);

iii) there exists $u(t)$ a continuous, positive and derivable function on $0<t\le 1$ with $u(0)=0$, having nonnegative derivative $u'(t)\in L([0,1])$, with $u'(t)\to\infty, t\to 0^+$ such that

$$|\varphi(t,x_1,y_1,z_1)-\varphi(t,x_2,y_2,z_2)|^2\le\frac{u'(t)}{Ku(t)}\min(|x_1-x_2|^2,|y_1-y_2|^2,|z_1-z_2|^2),$$

$$| h(y_1) - h(y_2) | \le \frac{u'(t)}{Ku(t)} | y_1 - y_2 |^2, \qquad (7)$$

for all $x_1, x_2, y_1, y_2, z_1, z_2 \in R, 0 \le t \le 1$, positive constant K and φ is any function a, b, f or g;

iv) with the same functions $u(t)$ as above,

$$| \varphi(t,x,y) |^2 \le u'(t) \min(1+|x|^2, 1+|y|^2, 1+|z|^2), \; \textit{and} \; | h(y) | \le (1+|y|^2), (8)$$

and X_0 is a finite random variable and Y_1 is given F_1-measurable random variable such that $E|Y_1|^2 < \infty$.

A similar forward-backward equation can be obtained using the canonical form.

Option pricing

The valuation of contingent claims is prominent in the theory of modern finances. Typical claims such as call and put options are significant not only in theory but in real security markets.

The option pricing model developed by Black and Scholes [3], formalized and extended in the same year by Merton [21], enjoys great popularity. In [24] we give a McShane version of this model. A similar result was given few years later by Sontea and Stancu in [37]. An adequate and complete version was given in [32] and some parts of this version will be presented in the following.

We consider a Black-Scholes market $M_{BS} = (S, B, \phi)$ (see [10],[14]) where:

i) $S = \{S_t\}$, $t \in [t_0, T]$, $t_0 \ge 0$ is the price process of a stock and we suppose that it satisfies the following differential stochastic equation by McShane type:

$$dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dz_t + \rho(t, S_t)(dz_t)^2, (9)$$

where

$$\mu(t, S_t) = \frac{-b}{1-\beta}\frac{S_t}{t}, \quad \sigma(t, S_t) = ct^b S_t^\beta, \quad \rho(t, S_t) = \frac{\beta}{2}c^2 t^{2b} S_t^{2\beta-1}, (10)$$

with $-1 < b < 0$, $0 < \beta \le 1$, $c \in R$;

ii) $B = \{B_t\}$, $t \in [t_0, T]$ is the price process of a bond and we consider that it satisfies the differential stochastic equation by McShane type:

$$dB_t = rdt + l(dz_t)^2; (11)$$

iii) ϕ is a trading strategy (see [22]) i.e. a pair $\phi = (\phi^1, \phi^2)$ of progressively measurable stochastic processes on the underlying probability space (Ω, F, P).

It is known (see [22]) that a trading strategy ϕ over the time interval $[t_0, T]$ is self-financing if its wealth process $V(\phi)$, which is set equal

$$V_t(\phi) = \phi_t^1 S_t + \phi_t^2 B_t, \; \forall \, t \in [t_0, T]$$

satisfies the following condition

$$V_t(\phi) = V_{t_0}(\phi) + \int_{t_0}^{t} \phi_u^1 dS_u + \int_{t_0}^{t} \phi_u^2 dB_u, \; \forall \, t \in [t_0, T]$$

where the integrals are understood in the McShane sense.

In [32] is proved the following results (using classical method of PDEs)

Theorem 1. *The arbitrage price at time* $t \in [t_0, T]$ *of the European call option with expiry date* T *and strike price* K *in the Black-Scholes market is given by the formula*

$$C_t = c(S_t, T-t), \forall\, t \in [t_0, T], (12)$$

where the function $c: R_+ \times [t_0, T] \to R$ *for* $t_0 \ge 0$ *and the function* $c: R_+ \times [t_0, T] \to R$ *(* $t_0 \ge 0$*) given by the formula*

$$c(s,t) = D(A(t)s - K)e^{B(t)s^{\alpha} + C(t)}, (13)$$

and $A, B, C: [0, T] \to R$ *are some continuous functions,* D *is a positive constant and* $\alpha = -2\beta$.

Zero-bond pricing

Let T^* be a fixed horizon date for all market activities. A *bond* is a contract, paid for up-front, that yields a known amount on a known date in the future, the *maturity* date, $T \le T^*$. The bond may also pay a known cash divident (the *coupon*) at fixed times during the life of the contract. If there is no coupon the bond is known as *zero-coupon bond*.

In the financial analysis sense, by a zero-coupon bond (or a *discount bond*) of a maturiy T we mean a financial security paying to its holder one unit of cash at a prespecified date T in the future. This mean that, by convention, the bond's principal (known also as *face value* or *nominal value*) is one dollar.

We assume throughout that bonds are *default-free*, that is, the possibility of default by the bond's issuer is excluded. The price of a zero-coupon bond of maturity T at any instant $t \le T$ will be denote by $B(t,T)$; it is thus obvious that $B(T,T) = 1$ for any maturity date $T \le T^*$. Since there are no other payments to the holder, in practice a discount bond sells for less than the principal before maturity - that is, at a discount (hence the name). This is because one could no incetive to invest in a discount bond costing more than its face value.

We consider a financial market analogous to Black-Scholes type (see [22]) with as financial underlying asset a zero-coupon bond with a fixed maturity date $T \le T^*$. It is known that the price of such underlying assets, denoted with $B(t,T)$, can be compute using its interest rate (see [22]).

Most traditional stochastic interest rate models are based on the exogenous specification of a *short-term rate of interest*. We write r_t to denote the *instantaneous interest rate* (also referred to as a *short-term interest rate*, or *spot interest rate* for borrowing or leading prevailing at time t over the infinitesimal time interval $[t, t+dt]$. In a stochastic setup, the short-term interest rate is modelled as an adapted process, say r, defined on a filtered probability space (Ω, F, P) for some $T^* > 0$. We assume throughout that r is a stochastic process with almost all sample path integrable on $[0,T]$ with respect to the Lebesgue measure.

We suppose that the interest rate r is gouverned by a stochastic differential equation by the McShane type of the following form:

$$dr_t = \mu(t,r)dt + \sigma(t,r)dz_t + \rho(t,r)(dz_t)^2. \quad (13)$$

Pricing of a bond is technically harder than pricing an option, since there is no underlying asset with which to hedge (see [22]). In this situation only alternative is to hedge with bonds of different maturity dates. For this reason we setup a portofolio containing two bonds with different maturities, T_1 and T_2. The bond with maturity T_1 has the price P_1 and the bond with maturity T_2 has price P_2. We denote the value of this portofolio with V. Thus we have that

$$V = x_1 P_1 + x_2 P_2$$

Theorem 2. *In [25] we obtained that the equation of structure term for pricing on such bond it is*

$$P_t + (\mu(t,r) + \rho(t,r) - \sigma(t,r)(\lambda+\eta))P_r + \frac{1}{2}\sigma^2(t,r)P_{rr} = rP \quad (14)$$

with the final condition

$$P(T,r) = 1. \quad (15)$$

The first results on this frame were developed by R. Negrea in [25] an represent the McShane version of two classical models given by Merton (see [20] and [21]) and Vaiscke (see [38]).

In [27] using similar way, was analysed the model developement by F. A. Longstaff (see [14]) for the classical Itô case and we assume that the dynamic of the interest rate is gouverment by the following stochastic differential equation by McShane type

$$dr_t = -a\sqrt{r}dt + \sigma\sqrt{r}dz_t + l(dz_t)^2 . \quad (16)$$

which is in the canonical form for $l = \frac{\sigma^2}{4}$. We take $\mu(t,r) = -a\sqrt{r}$, $\sigma(t,r) = \sigma\sqrt{r}$ and $\rho(t,r) = l = \frac{\sigma^2}{4}$ in the equation of structure term (14) and we have

$$P_t + [-a\sqrt{r} - \sigma\sqrt{r}(\lambda+\eta) + \frac{\sigma^2}{4}]P_r + \frac{\sigma^2}{2}rP_{rr} = rP, \quad P(T,r) = 1. \quad (17)$$

The basic idea was to search a solution by the following form

$$P(\tau,x) = e^{A(\tau)x^2 + B(\tau)x + C(\tau)}, \quad (18)$$

then we obtain

$$\begin{cases} A(\tau) = \dfrac{\sqrt{2}\tanh[\dfrac{-\sqrt{2}\tau\sigma^2 - 2\sqrt{2}}{2\sigma}]}{\sigma} \\ B(\tau) = -\cosh[\dfrac{\tau\sigma}{\sqrt{2}}]^{-\frac{2(a+\sigma(\lambda+\eta))}{\sigma^2}} + \dfrac{A(\tau)}{a+\sigma(\lambda+\eta)}\dfrac{\sigma^2}{2} \\ C(\tau) = \int_0^\tau (\dfrac{\sigma^2}{4}A(s) + \dfrac{a+\sigma(\lambda+\eta)}{4}B^2(s) - \dfrac{a+\sigma(\lambda+\eta)}{2}B(s))ds \end{cases} \quad (19)$$

(the explicite analytical expression for $C(\tau)$ was obtained but it is in the respect to Gamma and Hypergeometric functions and was omited to write here).

Now, using the above way, we consider a McShane stochastic evolution of short-term interest rate which extend the classical model of Cox-Ingersoll-Ross (see [11]), as following type:

$$dr_t = -ar_t dt + \sigma\sqrt{r}dz_t + l(dz_t)^2 . \quad (20)$$

In this case, the equation of structure term (14) become

$$P_t + (-ar + \frac{\sigma^2}{4} - \sigma(\lambda+\eta))P_r + \frac{1}{2}\sigma^2 r P_{rr} = rP . \quad (21)$$

We search a solution by the following form

$$P(\tau, x) = e^{A(\tau)x^2 + B(\tau)x + C(\tau)}, \quad (22)$$

then we obtain

$$\begin{cases} A(\tau) = \frac{1}{\sigma^2}(a + \alpha\tan[\frac{1}{2}(t\alpha - 2\arctan(\frac{a}{\alpha}))]) \\ B(\tau) = \frac{(-\frac{2\sqrt{2}a^2(\eta+\lambda)}{\alpha^2} - \frac{4\sqrt{2}(\lambda+\eta)\sigma^3}{\alpha^2} + 2\sqrt{2}\sigma\cos[\frac{1}{2}\alpha]}{\sigma\alpha} + \\ \frac{2\sqrt{2}\lambda\sigma\cos[\frac{1}{2}t\alpha)\sec[\frac{1}{2}t\alpha - \arctan(\frac{-a}{\alpha})]}{\sigma\alpha} \\ C(\tau) = \int_0^\tau (\frac{\sigma^2}{4}A(s) - \frac{\sigma(\lambda+\eta)}{2}B(s) + \frac{\sigma^2}{8}B(s))ds \end{cases} \quad (23)$$

where $\alpha = \sqrt{|-a^2 - 2\sigma^2|}$.

Also, in [27] was developed the model of L. T. Evans, S. P. Keef, J. Okunev (see [13]) for Itô case for the evolution of short-term interest rate and we assume an adequate equation in the McShane sense as the following form

$$dr_t = ae^{-kt}\sqrt{r}dt + \sigma e^{-kt}\sqrt{r}dz_t + le^{-2kt}(dz_t)^2 \quad (24)$$

In a similar manner as in the above example we search an explicite formula for the price of a zero-coupon bond by the form (14).

The functions $A(\tau)$, $B(\tau)$ and $C(\tau)$ result from the following differential system:

$$\begin{cases} A'(\tau) = \frac{\sigma^2}{2}e^{-2k(T-\tau)}A^2(\tau) - 1 \\ B'(\tau) = a - \sigma(\lambda+\eta)e^{-k(T-\tau)}A(\tau) + e^{-2k(T-\tau)}\frac{\sigma^2}{2}A(\tau)B(\tau) \\ C'(\tau) = e^{-k(T-\tau)}\frac{a - \sigma(\lambda-\eta)}{2}B(\tau) + e^{-2k(T-\tau)}\frac{\sigma^2}{8}(2A(\tau) + B^2(\tau)) \end{cases}$$

with initial conditions $A(0) = B(0) = C(0) = 0$.

We note that, in this case, the expressions for $A(\tau)$, $B(\tau)$ and $C(\tau)$ can be obtained but they are in respect to Bessel and Hypergoemetric functions and they were not given here. Naturally, if we replace parameters of model (a, σ, λ, η and k) with their estimations we obtain a simplified forms for this functions.

Conclusions

We proposed a model for the behavior of the financial assets on some unstable financial markets. The evolution on these markets is characterized by some "smoothed" life-time and some very "noises" life-time and this time periods are hard unexpected.

For these reasons, we propose an approach somehow, more general as there for a free financial market. Out study is just at the begining, but, as inthe example form above, the obtained results sustain our modeling for applications on the Romanian finacial market where the noise market is not a classical Gaussian noise, there exists more others random or non-random perturbations.

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