PRACTICAL MODELS FOR REPAYMENT OF CONSUMER CREDIT. CRITICAL ANALYSIS

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Abstract:
In practice handling a problem of consumption credit is based on the following two definitions of the fundamentals of cost reimbursement namely consumer credit cost and the rate or fraction to pay by a debtor to repay debt. Adopting certain models of repayment of consumer credit, the debtor repays actually less than he owes the lender. There are many other justifications bearing microeconomic or macroeconomic policy, and even for such practical situations. In this paper there are presented and analyzed some neoclassic models for repayment of consumer credit.

Key words: *credit cost, the rate or fraction to pay by a debtor, total present value of debt*

JEL classification: *D12, E51*

A case of short-term credit (less than a year) and medium (between 1 and 5 years) is so-called consumption credit granted to individuals mainly but not only them, for the purchase of consumer goods. Reimbursement is made by consumption credit year fractions.

Let V_0 the loan value at repayment start and d_{kl} the simple interest corresponding to the simple fraction (k,l) and to existing debt at the beginning of this fraction (fraction l of the year k) evaluated by single annual rate $p = 100 \cdot i$. As with any credit situation, the question of reimbursement of consumption credit is raised.

In practice handling a problem of consumption credit is based on the following two definitions of the fundamentals of cost reimbursement namely consumer credit cost and the rate or fraction to pay by a debtor to repay debt.

In the above context, the expression

$$D = \sum_{k=1}^{n} \sum_{l=1}^{n} d_{kl} \tag{1}$$

is called the cost of consumption credit, where *n* represents the number of years consumer credit gets reimbursed and *m* the number of fraction corresponding of one year.

According to previous definition, we have the following formula:

The cost of consumption credit = Amount of all simple outdated interests on fractions

The expression

$$S_F = \frac{V_0 + D}{n \cdot m} \tag{2}$$

is called rate on fraction or fractionality part of the consumption loan repayment.

As defined earlier, we have the following formula

$$\begin{array}{l}\textit{Fractionality part}\\ \textit{of the consumption}\\ \textit{loan repayment}\end{array} = \frac{\textit{Consumption loan} + \textit{Consumption loan cost}}{\textit{Year fraction reinboursement period}}$$

Based on previous definitions it results that the repayment of consumption credit is equal to the redemption or equal installments. We have therefore the possibility of adapting a mixed model of reimbursement that goes away from that rigorous and general principles of reimbursement models presented so far.

REINBOURSEMENT PRACTICAL MODELS

According to previous definitions, we can have a classical refund model or a non-classical model (combining a refund with equal redemption to determine interests d_{kl} and implicitly the cost of credit with repayment with equal fractionalities imposed by relation (2)).

If repayment of consumer credit is posticipated and with equal redemption, then:

- The average cost of consumption is

$$D = \frac{m \cdot n + 1}{2 \cdot m} \cdot V_0 \cdot i = \frac{(m \cdot n + 1) \cdot V_0 \cdot p}{2 \cdot m \cdot 100} \qquad (3)$$

- Part-repayment rate is

$$S_F = \left[1 + \frac{m \cdot n + 1}{2 \cdot m} \cdot i\right] \cdot \frac{V_0}{n \cdot m} = \left[1 + \frac{m \cdot n + 1}{2 \cdot m} \cdot \frac{p}{100}\right] \cdot \frac{V_0}{n \cdot m} \qquad (4)$$

Demonstration of these formulas is as follows: if reimbursement has redemptions (Q_F) equal on fractions then

$$Q_F = Q_{kl} = \frac{V_0}{n \cdot m},\ 1 \le k \le n,\ 1 \le l \le m$$

and thus giving up to the double indexing, the simple interest d_s on the fraction s, $1 \le s \le n \cdot m$, corresponding to the existing debt earlier this fraction, is

$$d_s = [V_0 - (s-1) \cdot Q_F] \cdot \frac{i}{m} = \left[1 - \frac{s-1}{n \cdot m}\right] \cdot \frac{i}{m} \cdot V_0$$

By summing interests d_s on s, we deduct the total interest as previously defined or the cost of consumption credit is

$$D = \sum_{k=1}^{n} \sum_{l=1}^{n} d_{kl} = \sum_{s=1}^{n \cdot m} d_s = \frac{n \cdot m + 1}{2 \cdot m} \cdot i \cdot V_0$$

That is precisely (3). Given (2) and (3), (4) follows immediately and the formulas (3) and (4) are shown.

If $m = 12$ then the repayment follows monthly and noting t_{12} the number of months as it takes for reimbursement, we deduce that if repayment is monthly posticipated and with equal redemption then:

$$\begin{array}{l} \textit{Cost of monthly} \\ \textit{posticipated reinboursed} \\ \textit{consumption credit} \end{array} = \frac{\textit{Credit} \cdot \begin{array}{c}\textit{year}\\ \textit{percentage}\end{array} \cdot \left[\begin{array}{c}\textit{repayment period}\\ \textit{in months}\end{array} + 1 \right]}{100 \cdot 12 \cdot 2}$$

or formalized, the cost of consumption credit repayable monthly posticipated is:

$$D = \frac{V_0 \cdot p \cdot (t_{12} + 1)}{2 \cdot 12 \cdot 100} \tag{5}$$

and therefore the annual rate of reimbursement S_F is:

$$S_F = S_{12} = \frac{V_0 + D}{t_{12}} = \frac{1 + \frac{(t_{12} + 1) \cdot p}{2 \cdot 12 \cdot 100}}{t_{12}} \cdot V_0 \tag{6}$$

Not to be confused that the interests d_{kl} given by relation (3) correspond to equal redemption payments for which the variable functionality in the classical system is given by computing the expression

$$S_{kl} = \left[1 + \frac{(n - k + 1) \cdot m - l + 1}{m} \cdot i \right] \cdot \frac{V_0}{n \cdot m} \tag{7}$$

with the fact that the same interests are composing equal rates (4).

With relations (3) and (4), very often in practice, we encounter a non-classical reimbursement model. Immediately we deduce, by direct calculation, that the present value of all rates (4) of a consumer credit valued at the beginning of the first fraction of reimbursement is equal to

$$A = \frac{2 \cdot m + (m \cdot n + 1) \cdot i}{2 \cdot m \cdot n \cdot i} \cdot \left[1 - \left(1 + \frac{i}{m} \right)^{-mn} \right] \cdot V_0 < V_0 \tag{8}$$

While the actual value of data (7) necessarily coincide (on a fundamental relationship of depreciation that the debt is equal to the sum of discounted future payments) to debt to be repaid V_0 (which can be verified easily by calculation), we find the inequality (8) to be seen and understood correctly. This inequality shows that adopting the model (3) - (4) of repayment of consumer credit, the debtor repays actually less than he owes the lender.

A question naturally arises: What is the interest of the creditor, in particular of a dealer for such a deal? ".

One of the typical responses of trade is "better than nothing."

There are many other justifications bearing microeconomic or macroeconomic policy, and even for such practical situations.

Example: Buying a consumer good, Mr. X has paid the amount $V_0 = 240\ 000$ m.u. in 24 equal installments, with an annual percentage $p = 9\%$. Specify the monthly rate in question if redemption takes place in the form (3) - (4).

In the relationship (3) we have

$$\text{The cost of credit} = \frac{240\ 000 \cdot 9 \cdot (24+1)}{100 \cdot 12 \cdot 2} = 22\ 500 \text{ m.u.}$$

and therefore, in the relationship (4) we deduce

$$\text{Monthly installement} = S_F = \frac{240\ 000 + 22\ 500}{24} = 10\ 937{,}5 \text{ m.u.}$$

According to relation (8) we obtain that the total present value of debt repaid by the amortization model (3) - (4) of a consumer credit is

$$A = 10\ 937{,}5 \cdot \sum_{s=1}^{24} \left(1 + \frac{0.09}{12}\right)^{-s} \cong 239\ 415 < 240\ 000 \text{ m.u.}$$

Given the relationship (8) and especially the influence of time factor (devaluation, different risks) it can be easily found that the convenient repayment process represented by relations (3) and (4) cannot be applied for long-term loans, in general, regardless of the nature of these loans.

Example: An appropriation of one billion m.u. is to be repaid in 10 years with a constant annual percentage $p = 10\%$, with equal redemption rates and posticipated payments. Specify annual reimbursement rates for the classical model as well as for the model (3) - (4) specific to a consumer credit and the loss that is if the creditor accepts the model (4) of the annual rate.

In the classical system, we deduct the annual rates, in order, 200, 190, 180, 170, 160, 150, 140, 130, 120 and 110 million u.m, whose total current value is one billion.

In the relations (3) - (4) follows a steady annual rate of 155 million m.u. and the total present value of these rates is

$$A = 155\ 000\ 000 \cdot \frac{1 - 1{,}1^{-10}}{0{,}1} = 952\ 408\ 500 \text{ m.u.}$$

thereby rendering a credit loss equal to

$$V_0 - A = 47\ 591\ 500 \text{ m.u.}$$

If consumer loan repayment rates is posticipated by fractions or equal fractionalities, then, when the conditions present in the definitions of the cost of consumer credit and the fractionality rate to pay by the debtor to repay the debt, we have:

- The cost of consumer credit

$$D = \left[\frac{n \cdot i \cdot \left(1 + \frac{i}{m}\right)^{n \cdot m}}{\left(1 + \frac{i}{m}\right)^{n \cdot m} - 1} - 1\right] \cdot V_0 = \left[\frac{n \cdot i}{1 - \left(1 + \frac{i}{m}\right)^{-n \cdot m}} - 1\right] \cdot V_0 \qquad (9)$$

- Rates of part-repayment

$$S_F = \frac{\left(1+\frac{i}{m}\right)^{n\cdot m}}{\left(1+\frac{i}{m}\right)^{n\cdot m}-1}\cdot\frac{i}{m}\cdot V_0 \tag{10}$$

The results are immediate if we take into account the interests on fractions given by the relations

$$d_{kl} = \frac{\left(1+\frac{i}{m}\right)^{n\cdot m}-\left(1+\frac{i}{m}\right)^{(k-1)\cdot m+l-1}}{\left(1+\frac{i}{m}\right)^{n\cdot m}-1}\cdot\frac{i}{m}\cdot V_0,\ 1\le k\le n,\ 1\le l\le m$$

whose summation gives the cost of credit in relation (9). Given (2) clearly this implies (10) and the demonstration is complete.

We notice immediately that the constant posticipated rate (10) is quite the constant fractionality from the model for clasic reimbursement (2).

Using the model of reimbursement (3) - (4), frequently encountered in practice, there are two immediate explanations, namely:

a) a connection with a relative ease of calculation;

b) a second about the fact that the debtor owes less than repaid (and even if his win is not large, it exists).

Remains to be seen, on a case by case basis, what interests, on a short term or a long term, directly or indirectly, is the creditor who adopt this model having!

Example: Mr X has acquired an electronic device for a total of 864 000 m.u. which is then repaid by 36 equal monthly posticipated installments evaluated by the constant annual percent of $8{,}5\%$. We need to determine the annual rate according to the model (3) - (4) and the model (9) - (10) and then to specify the win of the buyer whether the first model of reimbursement is adopted.

Based on the model (3) - (4) we have

$$\text{Credit cost} = \frac{864\ 000\cdot 8{,}5\cdot(36+1)}{100\cdot 12\cdot 2} = 113\ 220\ \text{m.u.}$$

$$\text{Monthly installement} = \frac{864\ 000+113\ 220}{36} = 27\ 145\ \text{m.u.}$$

From the model (9) – (10) we obtain

$$\text{Credit cost} = \left[\frac{3\cdot 0{,}085\cdot\left(1+\frac{0{,}085}{12}\right)^{36}}{\left(1+\frac{0{,}085}{12}\right)^{36}-1}-1\right]\cdot 864\ 000 \cong 117\ 886{,}32\ \text{m.u.}$$

$$\text{Monthly installement} = \frac{864\ 000+117\ 886{,}32}{36} \cong 27\ 274{,}62\ \text{m.u.}$$

The win of the debtor in the hypothesis of adopting the first repayment model is

$$C = (27\ 274{,}62-27\ 145)\cdot\sum_{s=1}^{36}\left(1+\frac{0.085}{12}\right)^{-s} \cong 4\ 106{,}2\ \text{u.m}$$

Similar to the models (1) and (2) it may be presented some other reimbursement models of consumer credit departing from these relationships.

As the economical phenomena are developing very quickly, they are becoming more and more complex. Given this context real-time situations must be thoroughly analyzed for the optimal decisions should be made.

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