MODELS OF PUBLIC DEBT SUSTAINABILITY ASSESSMENT AND THEIR UTILITY

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Abstract:
Ensuring public debt sustainability is one of the main challenges that public indebtedness policy has to face on the long run, susciting both practicioners and theoreticians. In this paper we aim to present an overview of the main models for assessing public debt and, more generally, budgetary policy sustainability, briefly presenting their contents and the main disadvantages and problems arising from their practical application.

Key words*: public debt, sustainability, government borrowing constraint, intertemporal budget constraint, IMF model*

JEL classification: *F34, H63*

1. Introduction

One of the basic challenges that public indebtedness policy has to face on the long term is represented by its sustainability, or in other words to provide the possibility of ***constantly resorting to public indebtedness and to maintain it over time*****.**

The assessment of public debt sustainability has been debated for more than two decades and became the main concern in the context of the growing public debt recorded in the '80s and of the debt crises which it triggered. On the background of resumed accentuated ascending trend of public debt in the context of the current economic and financial crisis, and of the difficulties which some developed states such as Greece already deal with for paying their due debts, the issue of public debt sustainability becomes once again a current one.

Both the economic-financial theory and practice are interested in the issue of providing public debt and, generally speaking, budget policy sustainability, ***multiple assessment models*** being proposed in time, with a higher or lower degree of applicability. The improvement of public debt sustainability assessment models, by including new variables and the construction of new indicators, proved to be a continuous process, in accordance with the increased complexity of the environment in which state indebtedness occurs. Therefore, a main role in this direction was played by the international financial institutions (the International Monetary Fund and the World Bank), concerned with assessing the outcomes of the actions performed by member countries, especially from the category of developing countries with low income, when introducing the High Indebted Poor Countries Initiative and, more recently, the Multilateral Debt Relief Initiative, in order to achieve the Millennium Development Goals.

2. Models of public debt sustainability assessment - fundamental approaches

Usually, the models of public debt sustainability assessment presented in literature are based on ***two main approaches***.

I. The first one is based on the studies conducted by E.D. Domar [Domar, 1944] and states that, in order to provide the sustainability of public debt, its share of GDP should be, on the medium and long term, decreasing or constant. Therefore, the condition which has to be fulfilled to ensure ***the stability of public indebtedness*** or, in other words, to provide the premises necessary for it not to grow continuously, is that the ***interest rate for government loans should not exceed the economic growth rate (of GDP).***

To demonstrate this correlation we should start from the equation of government budget constraint, concisely illustrated through the relation:

$D_t = D_{t-1} + rD_{t-1} - S_t$ or $D_t = (1+r)D_{t-1} - S_t$ (1)

The size of public debt from year t (Dt) is equal to the size of debt from a previous period (Dt-1), in addition to the expenses with public debt interest rates (calculated according to a real interest rate r) and deducting the primary budget surplus (St), calculated as subtraction between ordinary budget revenues and budget expenditure, other than public debt interest expenses. If S<0 we shall deal with a primary deficit.

By using the ratio with GDP (noted with Y) and knowing that $Y_t = (1+g)Y_{t-1}$, where g represents the real economic growth rate, the equation may be expressed this way:

$$\frac{D_t}{Y_t} = \frac{(1+r)D_{t-1}}{(1+g)Y_{t-1}} - \frac{S_t}{Y_t} \qquad (2)$$

or, in sizes expressed as share of GDP:

$$d_t = \frac{1+r}{1+g} d_{t-1} - s_t \qquad (2')$$

Using the approximation (1+r)/(1+g)~1+r-g, it follows that:

$d_t = (1+r-g)d_{t-1} - s_t$ (3)

and $d_t - d_{t-1} = (r-g)d_{t-1} - s_t$ (3')

This last relationship allows us to understand the evolution of a country's public indebtedness depending on the interest rate on state loans and the growth rate of GDP. The conclusions are given in Table 1.

Table 1: The dynamics of public debt depending on the interest rate, the growth rate of GDP and the primary budget balance

g-r / s	s<0 (primary deficit)	s>0 (primary surplus)
g-r>0 (strong economic growth)	public debt will converge to a stable level d*, where d*>0	public debt will converge to a stable level d*, where d*<0 (public savings)
g-r<0 (slow economic growth)	public debt will increase indefinitely, without converging to a stable level	undefined situation

a. If g>r and s<0, namely if the real economic growth rate exceeds the real interest rate on government loans and the government records primary deficits, then public debt as percentage of GDP tends towards a stable level (d*), being therefore, sustainable. The level of d* is obtained from relationship (3), in which we consider that $d_t = d_{t-1} = d^*$, and it follows that:

$$d^* = \frac{s}{r-g} \quad (4)$$

If the initial public debt is higher than s/(r-g), then it will have a descending trend in time, until it reaches the balance level, d*. Otherwise it will increase in time, until it reaches d*.

b. If g>r and s>0, in particular, the real economic growth rate exceeds the real interest rate on government loans and the state records a primary surplus, then public debt as percentage of GDP shall decrease and tends towards a stable level (d*), and thus public debt is sustainable.

c. If g<r and s>0, that is, the real economic growth rate is lower than the real interest rate on government loans and the state records a primary surplus, then public debt might be sustainable if the degree of initial public indebtedness complies with the relation $d(r-g)\leq s$. In this situation, the outcome of cutting down public debt as percentage of GDP due to practicing primary surpluses is more significant than the outcome of increase due to an interest rate which is higher than the economic growth pace, and on the medium and long term, the degree of public indebtedness decreases indefinitely. If not, the outcomes are reversed and public debt increases for an indefinite time, becoming no longer sustainable.

d. If g<r and s<0, namely, if the real economic growth rate is lower than the real interest rate on government loans and the state records primary deficits, public debt increases without limitations, thus becoming unsustainable.

By summarizing what we have stated so far, we can draw the conclusion that public debt is always sustainable if g>r, even if there is a primary deficit, therefore ***the condition that the economic growth pace should exceed the interest rate on government loans is enough to make sure that public debt does not increase indefinitely.***

From another perspective, the condition of maintaining the degree of public indebtedness at its current level or of diminishing it in time is fulfilled provided that the initial budget surplus is higher than the one required for providing stability, as it follows:

$$d(r-g)\leq s \quad (5)$$

One of the main shortcomings of the approach based on maintaining the stability of public debt is that it does not take into account the interdependencies existing between the elements involved, namely the interest rate, the volume and the structure of public expenses, the degree of public indebtedness and the economic growth rate. According to C. Sardoni [Sardoni, 2008], a significant share of productive expenses from all public expenditure may have beneficial outcomes on the economic growth rate and accordingly, on the share of public debt from GDP. Furthermore, this model does not target a certain degree of public indebtedness as being sustainable, but to maintain it steady, basically, at any level. However, stabilizing public debt at a high level would make a country vulnerable to shocks.

II. The second approach, based on the studies of J.D. Hamilton and M.A. Flavin [Hamilton, Flavin, 1986], considers that the ***compliance with the intertemporal budget constraint*** represents the main criterion for assessing the sustainability of public debt. This constraint is satisfied if the ***present value of future primary surpluses covers the size of current public debt***, the condition to be met to ensure public debt sustainability therefore beeing:

$$d_t \leq \sum_{j=1}^{\infty}[\frac{k_{t+j}}{q_{t+j}} s_{t+j}] \quad (6)$$

where $k_{t+j} = \prod_{i=1}^{j}(1+g_{t+i})$ and $q_{t+j} = \prod_{i=1}^{j}(1+r_{t+i})$

On account of the fact that such an assessment of public debt sustainability is based on very long time forecasts, theoretically covering an infinite time horizon, it is

less useful for the practical purposes. In addition, just like in the case of the previous approach which aims at stabilizing public debt, it does not allow the identification of possible cash-related difficulties, mostly because the structural elements of public debt are not taken into account. In practice, another basic issue is that of choosing a proper interest rate: on short or long time, before or after taxation etc.

The two approaches are at the base of many models for assessing the sustainability of public debt and, more generally, of budget policy. They were developed taking into account the specific circumstances of each country, either by also taking into consideration other financing sources for the budget deficit (such as money financing or the supply of income from privatization) and other sources of public debt increase (by taking over the debts of some private companies and including them in the public debt), or by including other factors of influence on the degree of public indebtedness, together with the interest rate and the GDP growth rate, such as the evolution of the exchange rate which is significant especially in the case of countries with large foreign currency public debt.

Nevertheless the assessment of public debt sustainability did not limit to the traditional approaches based on the stabilization of the public debt and on intertemporal budget constraint, but it evolved throughout time, towards ***other, more comprehensive conceptions***. Some authors consider other indicators as well and some define „signaling thresholds" in order to assess the situation of a country's public debt, in particular, in the developing and low income countries. In particular, in the recent years, the noted tendency is to include in the sustainability assessment models not only one indicator, but a group of indicators or a set of „thresholds".

3. The IMF model for assessing public debt sustainability

A reference model in assessing public debt sustainability was, throughout time, the model proposed by the ***International Monetary Fund.*** It was initially based on the guidelines established by Hamilton and Flavin, in 1986. Thus, it was considered that public debt is sustainable if „it satisfies the present value budget constraint without a major correction in the balance of income and expenditure given the costs of financing it faces in the market" [IMF, 2002]. Solvability was used as a synonym for sustainability, meaning that ***the present value of the current and future primary expenses is not higher than the present value of the present and future income, net of any initial indebtedness***. A simple and practical formula was established to assess solvency [IMF, 2002]:

$$\sum_{i=0}^{\infty} \frac{E_{t+i}}{\prod_{J=1}^{i}(1+r_{t+J})} \leq \sum_{i=0}^{\infty} \frac{Y_{t+i}}{\prod_{J=i}^{i}(1+r_{t+J})} - (1+r_t)D_{t-1} \quad (7)$$

Where :

$\sum Et+i$ is the sum of current and future primary expenditure;

$\sum Yt+i$ is the sum of current and future revenues;

D is the initial public debt stock;

$\prod(1+r_J)$ is the product of the discount rates of revenues and expenditure

Subsequently, other indicators based on calculating net present values have been considered in order to assess public debt sustainability. One of these compares the ***present value of public debt interest expenses with the present value of future primary budget surpluses*** and assumes that public debt is sustainable if the primary surpluses cover the interest-related expenses [IMF, World Bank, 2003]. Other authors, having analyzed the same indicator [Cline, 2003], consider that the primary budget surpluses

should also allow the generation of additional financial resources, in addition to those necessary to cover the interest-related expenses, in order to ensure and allow economic growth. Moreover, the ***comparison of the tax revenues to the public debt service*** was considered as an alternative measure and, in certain situations, a better sustainability indicator [IMF, 2003a].

The general approach for the evaluation and definition of debt sustainability as the result of comparing the present value of future payments and revenues or as net present value has been enhanced to include additional indicators: "*Instead of proposing a one-dimensional measure of debt sustainability, (...) such assessments should be informed by a menu of indicators, including the NPV of debt and debt service, relative to exports, revenues and GDP, and their evolution over time under realistic macroeconomic assumptions"* [IMF, 2003b]. The same criterion was retained and subsequently confirmed in a joint work of the International Monetary Fund and the World Bank "*Debt sustainability can be assessed on the basis of indicators of the debt stock or debt service relative to various measures of repayment capacity (typically GDP, exports, or government revenues)"* [IMF, World Bank, 2004]. In later developments, the need for institutional development and monitoring is also accepted as a component of the sustainability analysis.

Currently, ***the public debt sustainability is assessed***, within the analyses conducted by the International Monetary Fund personnel, ***based on two distinct models, one used for the developing and low income countries and the second model for the emerging economies and industrialized countries.*** The delimitation between the two categories considers the particularities noted with regard to public indebtedness: if the first one mainly resorts to concessional public loans, from external sources, so that, most often there are not any size-related substantial differences between the public and the external debt, in the second state category, both privates and governments have access to loan resources on the internal and external market and the sustainability analysis must be conducted distinctively for the public debt and the external debt, respectively.

The current analysis framework of the public debt sustainability for the ***low income countries*** has been approved by the International Monetary Fund and the World Bank in April 2005, revised in April 2006 and in November 2006. Its fundamental target is to guide the relevant authorities in the said states in adopting decisions on public indebtedness, as to ensure procurement of necessary financing resources under the terms of ensuring current and future reimbursement capacity, considering the circumstances specific to each country. The conducted sustainability analyses consist of:

1. Analysis of projections on public debt of a country for the following 20 years and of its vulnerability to external and political shocks – within basic scenarios and sock scenarios;
2. Assessment of the risk of public debt problems occurrence in the considered timeframe;
3. Providing recommendations on conceiving indebtedness strategies limiting the risk of such difficulty occurrence.

For the assessment of the public debt sustainability, proposed by the second pillar of the sustainability analyses, a group of indicators defined as maximum thresholds are considered, along with an assessment of the policies and institutions of each country (table 2).

The argument for policies and institutions assessment is that states with weak policies and institutions tend to have problems at lower levels of public debt than in the countries with strong policies and institutions. Using the Country Policy and Institutions

Analysis Index prepared by the World Bank, the countries are classified in three performance categories, namely: strong, medium and weak.

Table 2: Public debt thresholds for low-income countries

Quality of policies and institutions	***Indicators***				
	NPV of public debt in % of			*Public debt service in % of*	
	Exports	*GDP*	*Budget revenues*	*Exports*	*Budget revenues*
Low	100	30	200	15	25
Medium	150	40	250	20	30
High	200	50	300	25	35

Source: http://www.imf.org/external/pubs/ft/dsa/lic.htm

Depending on the performance category a country is part of, the maximum threshold of the indebtedness indicators differ, the indicator relevant to the countries with strong policies and institutions being the highest, following the fact that the risks involved are, in this case, more reduced. A public indebtedness indicator exceeding the maximum threshold suggests the risk that the respective country experiences a certain type of problem regarding the public debt. From this point of view, four risk categories can be identified:

- Low risk, when all indicators are below the threshold;
- Moderate risk, when the public indebtedness indicators are below the threshold in the basic scenario, but the stress tests indicate the fact that the thresholds could be exceeded, should external shocks occur or there are sudden changes in the promoted macroeconomic policies;
- High risk, when one or more public indebtedness indicators exceed the threshold in the case of the basic scenario;
- Public debt problems, when the country already registers difficulties in reimbursing the public loans and payment of the interests.

The analysis framework of debt sustainability for ***the emerging economies and industrialized countries*** (countries with access to international financial markets), unlike the framework for the low income countries, includes separate assessment of the total public indebtedness sustainability and of the total external debt (public and private), respectively. With regard to the public debt, 5 years projections are achieved on its relevance in the GDP, the requirement in order to ensure sustainability being that this indicator stabilizes or decreases within the considered timeframe. Moreover, in order to find the eventual problems regarding liquidity, the evolution of the financing gross need of the public sector is analyzed. The analyses are conducted within a basic scenario, supporting on the policies which the authorities intend to promote, and more alternative scenarios, based on stress tests.

The relationship that depicts the evolution of government indebtedness, depending on the specific factors of influence [IMF, 2008], is:

$$d_t - d_{t-1} = \frac{1}{(1+g+\Pi+g\Pi)}(\hat{r} - \Pi(1+g) - g + \varepsilon\alpha(1+\hat{r}))d_t - s_{t+1} \quad (8)$$

Where: - d is the public debt as % in GDP;

- s is the primary budget balance;
- $\hat{r}$ is the weighted average of the interest rates on domestic and external public loans;
- α is the proportion of the foreign currency public debt;
- π is the change in GDP deflator;

- G is the real GDP growth rate;
- ε is the variation of the foreign exchange rate (national currency/USD), when ε >0 the national currency depreciates.

Although complex, ***the analysis of public debt sustainability based on the methods currently used by the International Monetary Fund remains deficient from more points of view*** [Gray, Lim, 2008], of which one could note:

1. First of all, the public debt stabilization, as relevance in the GDP, does not always represent the best criteria to assess its sustainability and implicitly that of the budget policy. An increasing public debt does not necessarily involve that it is unsustainable if, for instance, a country promotes important budget deficits, making debts, in order to finance increased investment expenses or structural reforms, in order to sustain the economic growth.

2. Secondly, the main concern is to ensure public debt stabilization, without considering the level, either very high or low, at which such stabilization occurs.

3. Thirdly, the model does not appropriately consider the impact of the change in public debt structure (maturity, currency of denomination etc.) on its sustainability. For instance, the relevance of the public debt in the GDP does not change if a short term, foreign currency debt reduction is compensated by an increase of the long term debt, or national currency denominated debt, although the risk reduced following maturity increase.

4. Last but not least, the approach does not consider uncertainty or volatility in defining the macroeconomic parameters which the public debt sustainability is dependant upon. All indicators based on future values, projections, have more uncertainty sources: unforeseeable changes in the interest rate, economic growth rate, inflation, budget income etc.

Other problems faced in the use of the considered models are of methodological nature. One of them aims at the discount rate to be used in order to calculate the net present value of the public debt as well as the different roles the indicators on the public debt stock and service play.

4. Conclusions

The diversity of the approaches and models presented leads to the conclusion that ***there is no simple rule, generally applicable, in order to determine whether the public debt of a country is sustainable or not***. The issue of assessing public debt sustainability proves to be sensitive, both due to the theoretical divergences on defining the sustainability concept and on the economic and political factors diversity this is dependant upon, among which: the current size of the public debt and its structure, the primary budget balance, the taxation rate, the structure of the public expenses, volatility of the inflation rate, changes in the interest rate and exchange rate, volume of exports, the current account balance, the GDP growth rate, quality of institutions and promoted policies etc.

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