# SOLVENCY II: METHODOLOGICAL ASPECTS OF USING VALUE-AT-RISK FOR INSURANCE

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***Abstract:***
*Built on a three-pillar structure, the new European System of Insurance solvency "Solvency II" aims to ensure that insurance companies measure and manage better the risks they are expose. The consultation on the first pillar, on the quantitative requirements, largely refers to two well-known measures of risks: Value-at-Risk and Tail Value-at-Risk. The measuring of an insurance risk using the VaR technique claims certain adjustments. In this piece of work, we will present the context of VaR insurance application.*

***Key words:*** *Solvency, Insurance, Value-at-Risk, Tail Value-at-risk*

***JEL classification:*** *G22*

## 1. Introduction

According to the latest developments in risk management and given the recent developments in other financial sectors, European insurance field need to be reformed. The new European System of Insurance solvency "Solvency II" should adopt a risk based economic approach, to motivate the insurance and reinsurance companies to assess and manage their risks properly.

To allow insurance and reinsurance companies to meet their commitments to insurance policyholders and beneficiaries, member states should require those companies to set up adequate technical provisions. Actuarial and statistical principles and methods underlying the calculation of these technical provisions should be harmonized throughout the Community, to obtain a better comparability and transparency.

Supervisory regime should provide, firstly, a requirement for their funds which varies depending on risks, based on a prospective calculation, to ensure accurate and timely intervention of the supervisory authorities (the Solvency Capital Requirement), and secondly, a minimum level of security below which should not fall the financial resources (the Minimum Capital Requirement). Both capital requirements should be harmonized throughout the Community to ensure an appropriate protection degree for insurance policies holders.

The future system "Solvency II" provides that insurers must have an amount of technical provisions to enable them to meet their commitments with a probability of 75%. They will increasingly have a level of capital requirement allowing them to avoid ruin in one year horizon, with a very high probability of 99.5%.

Built on a three-pillar, system involves that insurance companies measure and manage better the risks they are expose. A good risk management will result in a lower requirement in terms of equity. The consultation on the first pillar, on the quantitative requirements, refers to two measures of risk which are well known: the Value-at-Risk

(VaR) proper for estimating the level of technical provisions with risk margin, and Tail Value-at-Risk (TVaR) proper for estimating solvency capital requirements.

## 2. Value-at-Risk and Tail Value-at-Risk

There are many measures of describing quantitatively the risk associated with holding a financial portfolio. All these measures are intended to measure the sensitivity of a financial asset with respect to a variation of a risk factor (rate of return and others). Among all measures, VaR is widely used in financial markets due to its relatively simple concept that allows its use by all managers.

The term "Value-at-risk" began to be used in the literature mainly from 1990s. However the origins of this measure seem to be much older in time. Mathematics underlying the concept of this value was developed especially in the context of efficient portfolio theory by Harry Markowitz. The emphasis which this theory puts on the market risk assessment involves the need to calculate the value-at-risk.

The Value-at-Risk is a probabilistic measure of the potential loss on a given horizon. If we consider a portfolio of assets X, then the Value-at-Risk for a period [0; t] with probability $\alpha$, denoted $VaR(X,\alpha)$, is defined as the quantile of order $\alpha$:

$$VaR(X,\alpha) = F_X^{-1}(\alpha),$$

where $F_X$ is the distribution function of the random variable of loss portfolio for the period [0; t] and $F_X^{-1}$ the inverse function.

The risk measure VaR is not a coherent risk measure[1] (see Acerbi (2004)). The subadditivity property is not verified by the VAR, if we consider two assets X and Y for which the estimated VaR for the same horizon [0; t] and the same threshold confidence $\alpha$, then the property of subadditivity

$$VaR(X+Y,\alpha) \le VaR(X,\alpha) + VaR(Y,\alpha)$$

is not always verified.

When using models that involve aggregation of risk, subadditivity is a desirable property. The presence of subadditivity implies that the aggregation of risks does not increase the overall risk.

A measure of risk which verified property of subadditivity is the Tail Value-at-Risk. The Tail Value-at-Risk at a confidence level $\alpha$ of a distribution X, denoted $TVaR(X,\alpha)$, is defined as the expected amount of loss beyond the VaR threshold:

$$TVaR(X,\alpha) = E[L_t / L_t \ge VaR(X,\alpha)],$$

where $L_t$ is the amount of loss for the period [0; t].

[1] A risk measure is called coherent if it satisfies four properties: the monotony, positive uniformity, transitivity and sub-additivity. P. Artzner, Application of Coherent Risk Measures to Capital Requirements in Insurance.

We can express the TVaR according to the probability density $f_t$ in the following manner:

$$TVaR(X,\alpha) = \int_{VaR(X,\alpha)}^{\infty} x f_t(x)dx.$$

We present below a graphic illustration for both VaR and TVaR risk measures.

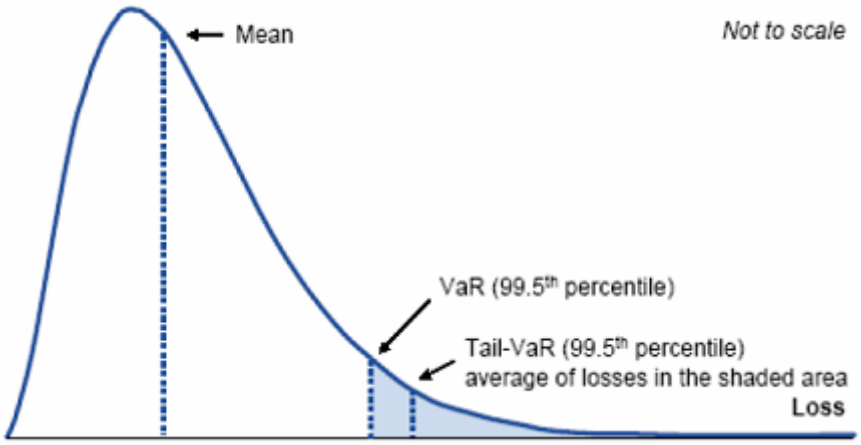


Figure1. VaR and TVaR illustration

Source : CEIOPS[2]

We note that the estimation of TvaR involves both the calculation of VaR and the estimation of the tail of the $L_t$ distribution. The most chosen methods to bring in practice for estimating the tail distribution are:

- Historical methods;
- Analytical method;
- Monte Carlo method;
- Bootstrap method.

A detailed presentation of estimation methods can be found in the Roncalli, T. (2004) and Denuit, M., Carpenter, A. (2005).

### 3. Adjustment of VaR method in Insurance

Financial management in insurance is a process whose main objective is to optimize the portfolio in compliance with regulatory constraints and commitments related liabilities. Effective financial management requires insurance companies to set a prudent and long term investment.

For the VaR estimation, insurer has only statistical material. The system "Solvency II" will provide a standard formula determined as much as possible on the basis of probability of ruin.

In general methodology of VaR calculation includes the following steps:

1) Risk factors identification;
2) Simulation of unfavorable company scenarios;
3) Estimation of tail distribution of the loss company's portfolio;
4) Determination of quantile of the loss density function.

[2] Committee of European Insurance and Occupational Pensions Supervisors – www.ceiops.org

Adaptation of technical banking to an insurer asset portfolio requires a number of adjustments, mainly to reflect the holding period (Fedor and Morel (2006) present quantitative analysis on this subject).

Portfolio management of an insurance company is characterized by a long term investment (buy and hold). An insurance company will invest in financial assets that guarantee long-term performance enabling it to meet its liabilities commitments to its policyholders and its shareholders. Under these conditions the portfolio of an insurance company is much more stable over time as a bank portfolio.

Consequently, insurance companies should try to estimate their market risk using VaR for horizons ranging from 3 months to one year. Therefore, the daily estimation of VaR in insurance is meaningless. Financial management in insurance have not the same objectives as in the banking sector.

## 4. Criteria for choosing a method to estimate VaR in Insurance

In this section we present criteria for choosing a method for estimating VaR (including the method: analytical, historical, Monte Carlo, Bootstrap) the most appropriate in insurance.

A first step in estimating VaR insurance is to check whether the daily rounds of all the risk factors affecting the solvency are or not identically and independently distributed (i.i.d). Each method for estimating VaR considers the time series of risk factors independently and identically distributed. If this condition does not verify, then we must correct the error specification data using ARCH and GARCH models.

We consider that the time series are (i.i.d). If more sets of risk factors are normally, we can calculate the VaR on horizon h calculating the daily VaR and then applying a technique of scaling by "root time"[3].

If the series of risk factors are normally and independently distributed, the VaR can be calculated with methods: Monte Carlo, analytic and historical method. Note that if the series of risk factors are indeed Gaussian, the three methods for estimating VaR are asymptotically equivalent. Note that if the series of risk factors are indeed Gaussian, the three methods for estimating VaR are asymptotically equivalent. In finite sample, we find fairly similar results.

If the series of risk factors are not normally and independently distributed only identically and independently distributed, we calculate the VaR on a daily basis with one of these methods: the Bootstrap method or the Monte Carlo method based on chronological series distribution of risk factors.

[3] The scaling is to multiply the day VaR by $\sqrt{h}$ , h is the horizon considered. Danielsson, J., Zigrand, JP (2006) On time-scaling of risk and the square-root-of-time rule.

## 5. Conclusion

The VaR models can provide insurance companies an effective tool for estimating the market risk of their portfolios for medium and long term. The "Solvency II" system provides for the determination of technical provisions to use VaR measure with a 75% probability and for the determination of capital requirements to use TVaR measure (which is a coherent measure of risk) with a probability of 99.5%. TVaR measure requires a methodology based on the theory of extreme values. However, in order that these methods of VaR and TVaR calculations are a reliable instrument for measuring insurance solvency, they require some adjustments. Major difference comes from the fact that the portfolio of insurance companies is characterized by a long-term investment, therefore it is necessary to use a time horizon between three months and one year. Choosing one of the methods of VaR calculation (analytical, historical, Monte Carlo, Bootstrap) depends on the statistical properties of used data.

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