USING MATHEMATICAL PROGRAMMING TO REDUCE THE INTERRUPTIONS IN THE TECHNOLOGICAL PROCESS TO THE PRODUCTION LINES IN INTERMITTENTLY FLOW

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Abstract:
In the production function, companies that produce large quantities of a single product are using a site-oriented product (production lines or flow assembly). Understanding the method of location and organization of these lines is essential to achieve a desired output with maximum efficiency.

Key words*: flow production lines, balancing flow production lines, heuristic method.*

JEL classification: *M 11*

Problem definition

Detailed programming of mass production is by default determined by the organization of the line. By grouping phases in operations and then materialize each operation as a workstation is actually deciding on how to move product throughout the manufacturing process, so by this is authorizing the fabrication of the line.

Programming mass production is usually indissolubly related with the problem of balancing flow lines.

It will lead the discussion on balancing flow lines on an example. Let’s admit that we have a technological process that has been split in 12 phases. Figure 1 shows each phase time and precedence-succession relations between phases as evidenced by technological reasons. The traced network after the nodes activity system has submitted the period of each phase over a node that represents it.

Phase F_i	**The immediately predecessor**	**Time** t_i
F1	-	6
F2	-	9
F3	F1	4
F4	F1	5
F5	F2	4
F6	F3	2

F7	F3,F4	3
F8	F6	7
F9	F7	3
F10	F5,F9	1
F11	F8,F10	10
F12	F11	1
Total time		55

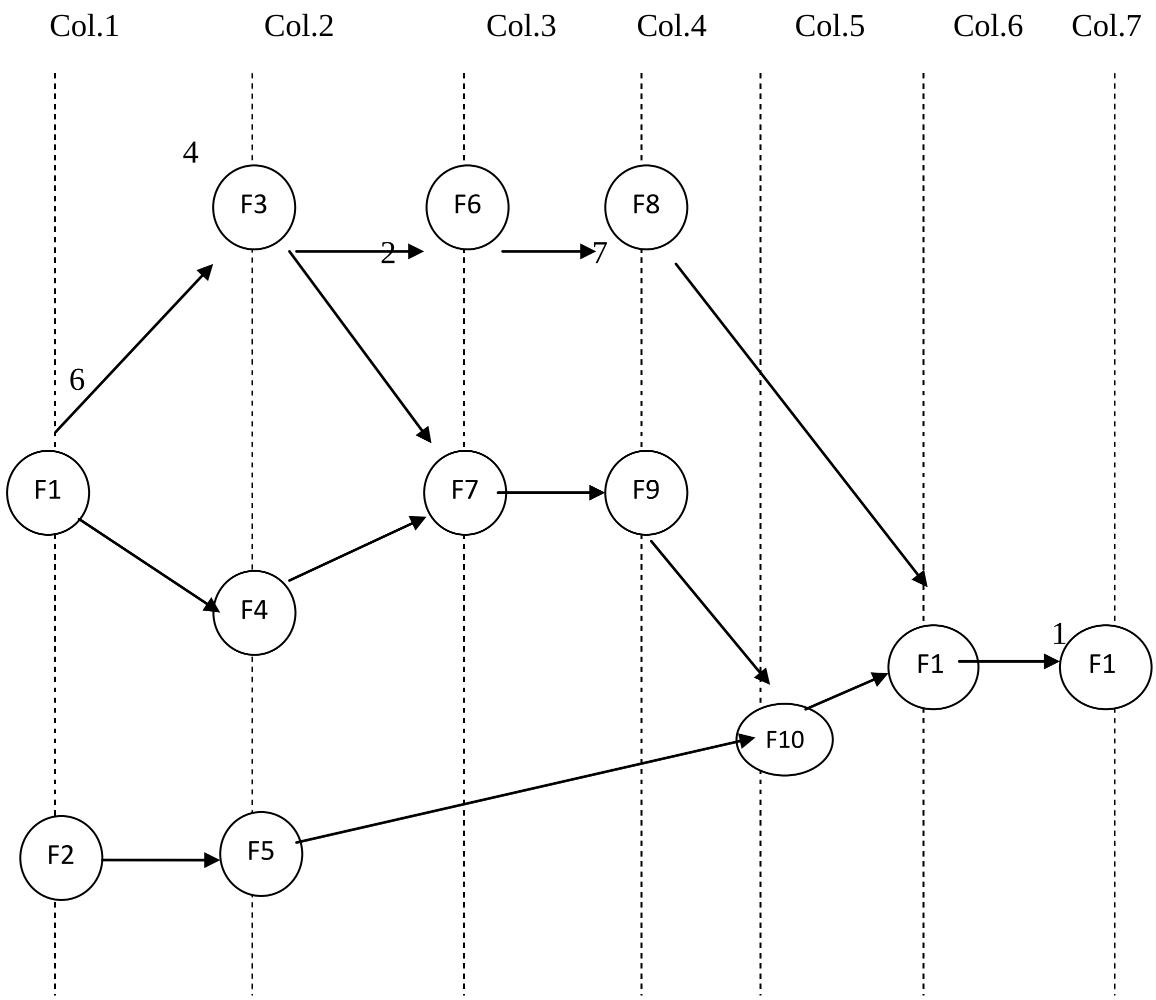

Figure 1. Technological process with 12 phases

What is desired is to organize a flow line on which to develop the technological process taken as example. For the reasons set out above we want the line to be as well balanced as to provide a minimum value of the product N•r, and also for $d_{\%}$.

Before tackling the balancing problem we are presenting a summary of notations:

r = line rhythm;

N = workstation number;

j = index that shows the phase;

F_j =stage number j;
J = total number of phases;
t_j = time of phase j;
i = index that shows the operation that takes place to the i - workplace;
p_i = operation time i;
d = total dead time of the line flow;
$d_{\%}$ = total dead time, expressed as a percentage from N•r;

For a unitary synchronization it means that: $r \geq \max p_i$;

In order to understand better the problem is useful to consider the link between rhythm and number of workplaces. Dead time d% which varies with N and r, present the fallowing family of hyperboles:

$$Nr = \left(\sum_{j=1}^{J} tj\right) / \left(1 - \frac{d\%}{100}\right) \qquad 1.1$$

Equation of this family of curves has d% as parameter and is derived from the relationship 3. Is to notice that, on account of continuing discussion, only to the case of unitary synchronization, we have r≥max and p_i ≥max t_j. Because max t_j=55; as a consequence we consider values only until 55. Fallowing minimization of dead time $_{d\%}$ we tend to choose N and r so that we are alwayas on the curve with $d_{\%}$=0.

However, this is not always possible because of the fact the N number of work stations must be entire. For a given rhythm r, minimum number Nmin of costs per line is:

$$N_{min} = \left(N \geq \frac{\sum_{j=1}^{J} t}{r}\right) \text{ with N integer number} \qquad 1.2$$

For example, if r=22 are required at least 3 work stations, in which case d%=16, 67%. Obviously, because of the requirement that N be integer, with r=22 can not achieve a perfect balance.

To reach that stage group in workstations that will lead to a minimum value of the product N•r may proceed in two ways:

Whether the rhythm value is fixed and is looking for the grouping with a minimum number of workplaces

Is predefined number of workplaces and are grouped so that the line rhythm to be minimum.

In general the alternative „a" is more used because almost always the line rhythm is required by the production task (plan) of the unit. For example, if 960 pieces must be made daily and the line will work in one shift, is necessary a rhythm r=30 seconds. It was noted that there is some flexibility in setting the rhythm. Thus, the same production of 960 pcs / day can be done with a line that works in two shifts, with the rhythm r=1 minute, or we can organize two identical lines, which operate in parallel in one shift, each one with a rhythm of 1 minute etc. In any way, we aim to establish such a rhythm that to be closer to one of the minimum points from the dead time curve.

But sometimes must be used alternative "b", such as in the case of rebalance of an existing line for the fabrication of a redesigned product. Admit that the old flow line is equipped with a numbers of machines set on foundations, which makes non economic changing the emplacement or redesigning it. In this case the number of workstations is taken from the old line, and after grouping the phases of the new technological process so that to obtain a minimum rhythm or, when disturbances appear in the functioning of a line like when a worker is missing or failure of a machine, the line must be rebalanced by phase reallocation of the workstation number given. As fallows we will concentrate on the first alternative of the balancing problem.

2. Solving the problem using mathematical programming

As there are a large number of ways in which "j" phases can be grouped into N workstations, balancing flow lines is clearly a combinatorial problem, which indicates us from the start that will be very difficult to solve. Literature presents a wide variety of models and ways of optimization for the problem of balancing lines; we remember binary programming methods, the dynamic programming, modeling under the shape of a flow in networks, methods of searching in trees, heuristic methods.

To be able to guess problem's nature, its large sizes and the difficulties of solving it will present here the modeling of balancing problem as a mathematical program with binary variables (variables 0-1). It even make reference to the example introduced above and we will impose a rhythm r=12. The balancing problem which requires to be solved is: as given the technological process in Figure 1 and a rhythm of 12 - group the stages of the process in the smallest possible number of workstations.

Are used the fallowing binary variables:

X_{ji}= 1 if the phase j is executed to workstation i;
0 if the phase j is executed to workstation i;

δ_i= 1 if the line contains the workstation with number i;
0 if the line does not contain workstations with number i.

Based on the previous relationship is calculated as the minimum required number of posts is N_{min}=5. Since is unknown, if in terms of the example set, can be obtain a line with 5 workstations and a rhythm equal 12. Though it is easy to group the 12 phases in 6 workstations, so that in our statement is not necessary to consider more; so j=1, 2...6

Now it writes the programming model with binary variables first in general form, and after are given explanations and are made particularizations for the given example.

$$MinZ=\sum_{i-1}^{L}\delta_i \quad 1.3$$

s.c. $$\sum_{i=1}^{L}X_{ji} \qquad j = 1,...,J \quad 1.4$$

$$\sum_{j=1}^{J}t_j * X_{ji} \le r \qquad i = 1,...,L \quad 1.5$$

$$\begin{cases} X_{k1}-X_{j2}\ge 0 \\ X_{k1}+X_{k2}-X_{j2}\ge 0 \\ \dots\dots\dots \\ X_{k1}+X_{k2}+\dots+X_{k,L-1}-X_{j,L-1}\ge 0 \end{cases} \quad K \in \{\text{Set of indices of immediate predecessors from phase j}\} \quad 1.6$$

$$\sum_{j1}^{J}X_{ji} \le J * \delta_i \qquad i = 1,...,L \quad 1.7$$

$$X_{ji} = 0 \; or \; 1 \qquad j = 1,...,J; i = 1,...,L \quad 1.8$$

$$\delta_i = 0 \text{ or } 1 \qquad i = 1,...,L \quad 1.9$$

As variables δ can be only 0 or 1, δ, object's function will be equal with the variables δ with the value 1, which represents exactly the number of workstation from the line; is required that this number to be minimized. The upper limit L of the sum from the relation 1.3 makes us understand the fact that in the final optimal configuration will not have mote than L workstations. For our example, as we mentioned above, we can find easily a group of 6 workstations, configuration towards the optimal solution cold possibly have fewer jobs, in no case more, as fallows the objective function is:

$$Z=\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6.$$

Relations (1.4) show that each phase should be included in one and appoint a single workstation. Thus, with reference to stage F1 can be written:

$$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} = 1$$

How any X_{ji} can be only 0 or 1 results from the written condition that only one x from 6 take value 1, namely the one corresponding to workstation which include phase F1.

The left member of the relation (1.5) summarizes the time of all grouped phases in the workstation i; indeed, if X_{ji} is 0 the time of the phase j is excluded from the summing up. Relation (1.5) says that none of the workstations can include phases summarized to exceed the line rhythm r. In particular for workstation 1 we write:

$$6X_{11} + 9X_{21} + 4X_{31} + 5X_{41} + 4X_{51} + 2X_{61} + 3X_{71} + 7X_{81} + 3X_{91} + 1X_{10,1} + 10X_{11,1} + 1X_{12,1} \leq 12$$

Conditions (1.6) model the relations of previous sequence between phases. We take for example, random phase j=7, identify the set of its immediate previous indices as being {3, 4}.

If phase 7 is executed, let's say, to workstation1, the predecessors F3 and F4, must necessarily make part from the workstation, thing that is equivalent to require X31=1 and X41=1. This can be expressed concisely as:

$$X_{31} \geq X_{71} \text{ si } X_{41} \geq X_{71}$$

If phase 7 is executed, but, to workstation2, than the predecessors F3 and F4 must make part of either workstation 1 or workstation2, that X31+X32=1 and X41+X42=1. But if F7 is contained in other workstation different the two restrictions must not be imposed. Again, it is written in short:

$$X_{31} + X_{32} \geq X_{72} \text{ and } X_{41} + X_{42} \geq X_{72}$$

Proceed similarly admitting any location of phase 7 in workstation 3, then in 4 and finally 5. It was noted that if phase 7 is included in the last workstation, the six, yhen any condition can be imposed over the predecessors F3 and F4 because, anyway, any stage is executed until the last workstation.

The general form of this restriction, considering the workstation's line from 1 until L-1 and after rearranging terms, it appears as group relations (1.6).

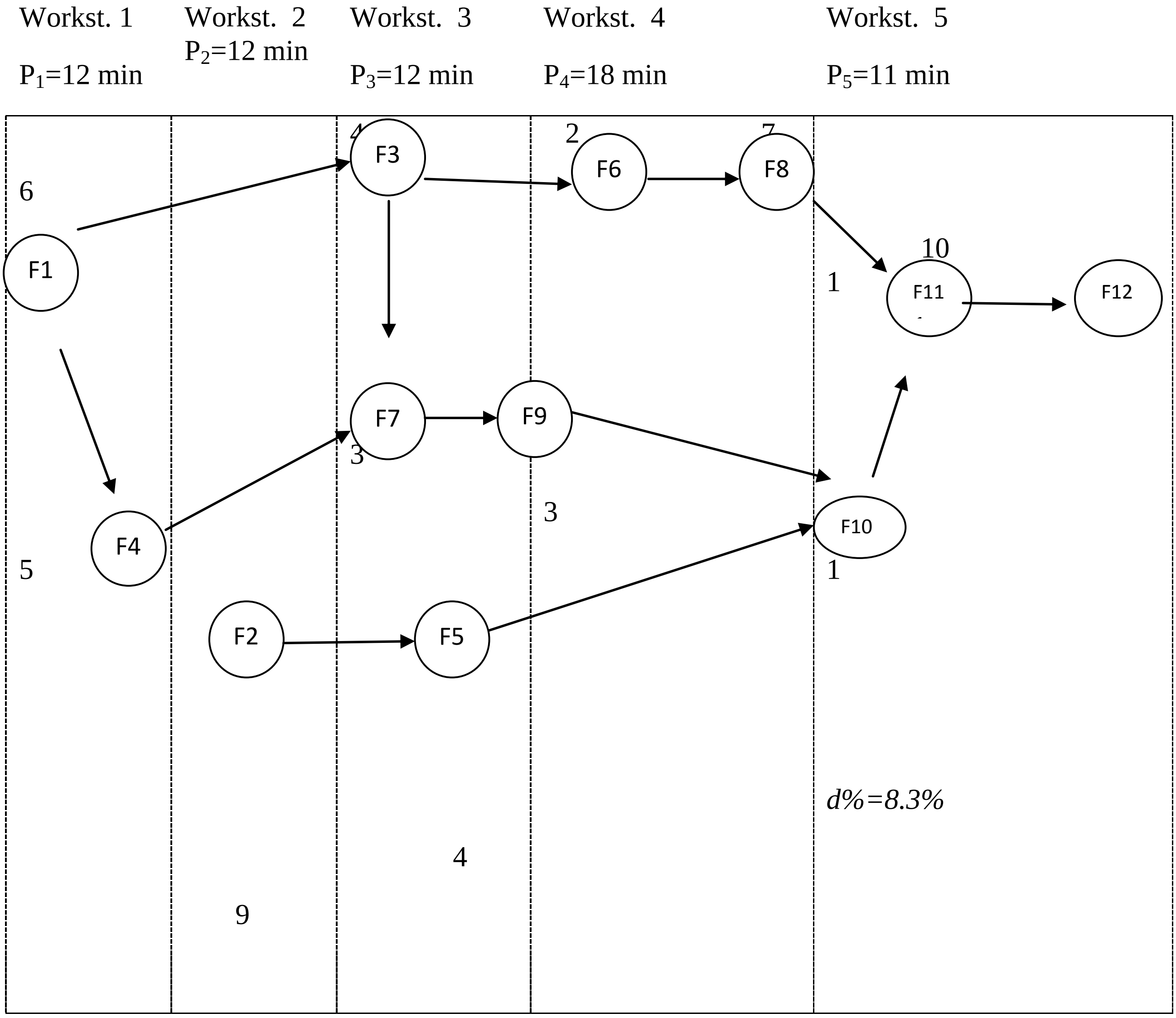

Figure 2. Optimal solution with five workstations

Was admitted L as upper limit of the workstation number from the line. It is conceivable that, eventually, the optimal configuration to have less than L workstations; so we need to ensure that no phase is included in any of hypothetical workstations that nor remain in the final configuration of the line. For example, to avoid that workstation 1 to be nonexistent (that (δ_1=0), while some stages will be still allocated we write: $X_{1,1}+X_{2,1}+X_{3,1}+X_{4,1}+X_{5,1}+X_{6,1}+X_{7,1}+X_{8,1}+X_{9,1}+X_{10,1}+X_{11,1}+X_{12,1}\leq 12\delta_1$

If δ_1=0, clearly that all X that have 1 as the second index should be zero; but if workstation 1 is operative, then the above inequality requires that in this should not be included more than 12 phases, which is actually not restriction since the total number of phases are 12.

Conditions of this type made for a general case with L workstations are formulated in several relationships.

Once with the model formulated, solving it will give the optimal values of the decision variables. For the problem considered by us as example there are several optimal solutions (optimal multiple), which groups the 12 phases in 5 workstations; dead time d%=8.3%. One of the solutions is:

$$\delta_1^* =1;\ \delta_2^*=1;\ \delta_3^*=1;\ \delta_4^*=1;\ \delta_5^*=1;\ \delta_6^*=0;$$

$$X_{11}^*=1;\ X_{22}^*=1;\ X_{33}^*=1; X_{41}^*=1; X_{53}^*=1; X_{64}^*=1;$$

$X_{73}^{*}=1;\ X_{84}^{*}=1;\ X_{94}^{*}=1; X_{10,5}^{*}=1; X_{11,5}^{*}=1; X_{12,5}^{*}=1.$

with all the other X_{ij}^{*} equal 0.

Figure 2 is a graphical representation of the line flow corresponding to the above solution.

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