SIGNIFICANCE OF MATHEMATICS FOR ECONOMICS

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Abstract:
Mathematics plays a very important role in economics. This role has been increasing in importance in last years. Mathematics is thus increasingly important in terms of the expression and communication of ideas in economics. This in itself is a matter of interest, particularly with respect to the public understanding of economics. Mathematics is increasingly significant for economics, namely its role in the economy itself. Increasingly activity in financial markets (particularly in derivatives trading) is governed by mathematical models. We will focus here, rather, on the effect of mathematisation on the content of economics. This is preceded by a brief account of the history of the role of mathematics in economics.

Key words: *axiomatisation, mathematisation of economics, econometrics*

JEL classification: C00 – *Mathematical and Quantitative Methods - General*

1. INTRODUCTION

The role of matematics in economics has been significant for almost a century, and has been increasing in importance particularly in recent years. A comparison of academic journals now with, say, fifty years ago reveals a tremendous increase in mathematical expression; Backhouse (1998) [2] reports an increase in the incidence of algebra in articles in the two leading economics journals from 10% in 1940 to 80% in 1990. The same is true also of textbooks at all levels.

Grubel and Boland (1986) [9] report a concern, identified through a questionnaire survey of leading economists, with the increasing use of mathematics in published research and graduate teaching. However the survey results also implied that emphasis on pure mathematics was a rational response to incentives within the profession. Concern with the extent of mathematical expression may derive from the view that it crowds out other modes of expression, i.e. the issue is over how ideas are *communicated*. But there is a more fundamental issue concerning the role of mathematics in economics, namely its potential effect on *content*. One interpretation is that economics has been undergoing technical change, employing more mathematics and more sophisticated statistical techniques, which has improved the productivity of the discipline; the change in content is thus one of undoubted improvement. But concerns have been raised that mathematisation has proceeded at the cost of attention to matters which cannot be expressed mathematically, ie the alternative modes of communication can actually allow analysis in areas closed to mathematics. The issue is thus the fundamental one of what we understand by the discipline of economics and what it can achieve. This issue too feeds back into the issue of the public understanding of economics *as a discipline.*

Mathematics is increasingly significant for economics in a third sense, namely its role in the economy itself. Increasingly activity in financial markets (particularly in derivatives trading) is governed by mathematical models. The importance of economists' role in developing these models is attested to by the award of the Nobel prize for economics to Black and Scholes for their ill-fated finance model. In what follows, we do not focus on this role of economics, other than to refer to it in illustration.

2. HISTORY OF MATHEMATICS IN ECONOMICS

Mathematics first took on a significant role in economics in the last century in the build-up to what is commonly referred to as the Marginalist Revolution. This was a period in which Classical concerns with production, growth, and the distribution of the fruits of growth among social classes, were being replaced by concern with market exchange. The focus thus shifted from the level of the economy and social classes to the level of the individual. Leon Walras, in particular, set out to establish the conditions for a successful co-ordination of market exchange, and he did so mathematically. Along with Augustin Cournot, he is responsible for the introduction of the systematic application of mathematics to economics.

At the same time, there was a concern that economics should be seen as a discipline on a par with the physical sciences (Drakopoulos, 1991) [7]. Walras's father, like many other economists of the time, saw mathematics as the vehicle for achieving this goal. Further, just as the physical sciences were being built up in axiomatic fashion on the basis of units of energy, etc, economics was being built up axiomatically on the basis of units of utility. The motivation of individuals in the economy engaging in market exchange is understood as the maximisation of utility, a human motivation which clearly lends itself to mathematical treatment. Walras (1965) [17] went further: "It is only with the aid of mathematics that we can understand what is meant by the condition of maximum utility". And indeed, the term "Marginalist Revolution" refers to the mathematical result of the marginal conditions for market equilibrium, as derived by calculus.

Such a development inevitably evoked a reaction. The Historical School in Germany (which provided the origins of the subsequent massive data-collection exercise in the US) argued that theory should emerge by induction from data, rather than being developed purely deductively. Their focus was thus on data collection rather than theory, mathematical or otherwise. The Austrian school, led by Carl Menger, took a deductivist approach, but deliberately shunned mathematics. The focus of their work was on the dynamics of the economic process, particularly the activities of entrepreneurs, rather than on market equilibrium. Their argument against the capacity of mathematics to assist such analysis was that humans are creative, purposeful beings, whose behaviour cannot adequately be represented deterministically. Further, their subjectivist approach led them to view published data as an inadequate reflection of the perceptions (rather than actuality) which prompted action. Here we have an important argument about the relationship between mathematics and the content of economics, to which we will return in the next section.

In the United Kingdom, the leading figure in the subsequent development of economics was Alfred Marshall. He was influential particularly in his efforts to promote economics as a discipline and to project it as a unified social science, in spite of the debates then raging between the mathematical pure theorists, the empiricists and the non-mathematical pure theorists. It is significant that his *Principles* (Marshall, 1890) [15] restricted mathematical reasoning to footnotes, so that the argument in the text was purely verbal. This is regarded as indicative of his views on the limitations of mathematics in economics; although he was more willing than the Austrians to engage in mathematical theorising, the focus of his research, like theirs, was on economic process rather than equilibrium. Further, his use of deductive reasoning (including his use of mathematics) was explicitly restricted to short chains of reasoning, ie partial analysis. Unlike Walras, he did not aspire to construct a complete mathematical system.

John Maynard Keynes was, like Marshall, trained initially as a mathematician, and also had reservations about its scope in economics. While he used mathematics to a limited extent, he argued that its capacity to capture the content of economics was limited, and thus so also was its application in empirical work. We will consider his arguments more fully in the next section. But his major impact on economics was to start off a new research agenda which gave particular impetus to the role of mathematics and its application in empirical work. Keynes provided the foundations for modern macroeconomics, which focuses on the economy in aggregate, rather than individuals. He also set out a policy agenda for government which required that the theory be tested and applied empirically. On this basis, increasingly elaborate mathematical models of the economy were constructed, aided by advances in computer technology and by the collection of data series, deriving from the ideas of the Historical School.

Methodological issues arose over the meaning of these aggregate models. In particular, the controversial suggestion was made by Friedman (1953) that predictive success should be the sole criterion for theory choice; theorists should not seek to explain, ie theories should not be regarded as representing causal processes. The form, as well as the extent, of mathematical representation, by implication, was secondary to empirical predictive success (although it was assumed that prediction would be based on mathematical models of some form).

But, by this stage, mathematics had in many ways taken over from physics as the model for the discipline of economics. This was evident in the force behind the further development of economics in the form of application of the principles of formal axiomatic systems. Macroeconomics had emerged as a mathematical system quite separate from microeconomics. Not only did they address different questions (failure of markets to co-ordinate and co-ordination success, respectively), but macroeconomics seemed to flout the axioms of individual behaviour on which microeconomics was founded. As a result, developments in macroeconomics over the last three decades can be understood as attempts to build up an over-arching general equilibrium system based on common axioms of individual behaviour.

Modern economics thus relies heavily on mathematics. But measurement problems, and more fundamental methodological problems, have created a bifurcation between pure theory and applied theory. While the former constructs sophisticated mathematical analysis of individual behaviour based on utility maximisation principles, with an emphasis on existence proofs, the latter focuses more on the reduced forms for which there are corresponding data. Given the different aims of the two activities, the mathematics employed in pure theory will thus tend to differ from that employed for the purposes of statistical testing. Backhouse (1998) refers to the increasing propensity to separate the ‘pure theory’ part of an investigation from the ‘empirical’ part even within individual articles.

The debate continues as to the extent to which this bifurcation between theory and application is problematic and whether it is surmountable. But it should be noted that this bifurcation is representative of what is called mainstream, or neo-classical, economics in the Walrasian tradition. Theory which has developed in the tradition of Menger, Marshall and Keynes makes much more limited use of mathematics on methodological grounds. Clearly this poses problems of communication within economics, in that the mainstream embraces mathematics as the preferred mode of expression, and indeed models itself on mathematics.

With this history in mind, we turn now to address more directly the methodological issues raised by the mathematisation of economics.

3. SIGNIFICANCE OF MATHEMATICS FOR ECONOMICS

3.1. Mathematics and Formalism

The role of mathematics in economics can usefully be discussed in relation to the role of formalism in economics. While the two terms are often used interchangeably (Krugman, 1998) [14], an argument need not be mathematical to be formal (see Chick, 1998) [4]. Further, it has been argued recently (by Backhouse, 1998) that, unlike mathematics, formalism entails the tighter condition of fixity of meaning. Weintraub (1998) has demonstrated the changing meaning of terms in mathematics. In particular, formalism also includes the notion of rigour; but scientific rigour may itself be subject to different meanings. Thus, while, at the turn of the century, scientific rigour referred to testing against empirical evidence, it is now associated more with mathematical axiomatisation.

We have noted above the greater extent of mathematical formalism in pure than in applied economics. But formalism also has consequences for empirical testing; it requires the notion of fixity of meaning applied to data too. This allows for reference to 'the facts' as objectively measured phenomena with fixed meaning independent of theory. Indeed Mirowski (1991) [16] argues that the very act of measurement imposes a mathematical structure. For example, the conventional market diagram presumes homogeneity of commodity space which is not in fact fixed in nature; Mirowski argues that the degree of homogeneity will vary depending on the changing social perception of market activities. Insofar as economics embraces formalism, therefore, it embraces a particular general approach to mathematics which derives from logical positivism and has implications at both the pure and applied levels. Within this general approach, there are then different uses made of mathematics depending on whether the research is pure or applied.

The benefits of formalism for economics are (Backhouse, 1998) clarifying what is known through demonstrating what can and cannot be proved:

- enabling a cumulative growth of knowledge since formal arguments may be readily understood by subsequent generations (note the necessity for fixity of meaning)
- providing an engine for discovery; a strong advocate of mathematisation of economics,

But, as Backhouse (1998) points out, the process of mathematisation itself may change the meaning of economic terms. He refers to the change from Adam Smith's notion of self interest brought about by its incorporation within a formal general equilibrium system; the social content of Smith's self-interest was lost in the atomistic axioms of general equilibrium theory. The application of formalism to the argument was represented as scientific progress by Arrow and Hahn (1971) [1]. Similarly, Keynes's theory of expectations under uncertainty changed meaning when formalised in the Rational Expectations Hypothesis. Formalisation inevitably eliminated Keynes's emphasis on unquantifiable risk. Yet Lucas (1980) represented this too as technological advance. The benefit of formalism in terms of promoting the growth of knowledge relies on meaning remaining unchanged.

The first benefit from formalism noted above is a paraphrase of an expression used by another advocate of formalism in economics, Frank Hahn. In response to Joan Robinson who saw mathematics as having only a limited role in economics, Hahn (1989) [9] would dismiss her arguments as things which 'cannot be said'; they lay outside the purview of the formal structure, ie they could not be demonstrated to be true within that structure. Thus an important issue of whether the change of meaning in the formalisation of Smith's self interest or Keynes's expectations under uncertainty eliminates something important which nevertheless 'cannot be said'. This brings us back to the arguments referred to in the previous section about the limitations to the scope for the mathematisaiton of economics.

The critics of mathematisation based their critiques on their understanding of the subject matter of economics. This represents a significant methodological departure from the more general trend we had identified of mathematisation itself being a guiding principle in the drive to establish economics as a science. Whether the subject matter of the physical sciences is amenable to the kind of logical positivism which has dominated economics is a question in itself. But the question is clearly a significant one for a social science where the objects of study are creative, purposeful, social beings who act within an evolving institutional environment. How far can human behaviour be represented as conforming (albeit stochastically) to deterministic principles? While the issue is most stark when formalist economic reasoning is applied to highly personal matters, such as the family, it has general application to all human activity which has economic content.

Drawing inspiration from Keynes, Chick (1998) [4] argues that real economies are open, organic systems which cannot be fully understood by means of closed, formal theoretical systems, and the mathematics of general equilibrium systems requires closure, and it requires that interactions between the units of analysis to be deterministic, ie it requires atomism. While formalism in the sense of rigour, precision and clarity of reasoning is a necessary feature of science, formalism in the sense of mathematical expression is not. One of the perceived virtues of mathematics as noted above is its precision; yet Keynes, following Marshall, pointed out the virtues also of vagueness:

> *"Much economic theorising to-day suffers, I think, because it attempts to apply highly precise and mathematical methods to material which is itself much too vague to support such treatment"* (Keynes, 1998) [13]

The Black-Scholes episode provides a pointed illustration of this argument. Mathematical models of risk in financial markets require that all risk is quantifiable. But the financial crisis which caused the Black-Scholes system to collapse was not amenable to frequency distribution analysis - but nor was it random - so it lay outside the model.

The general issues to be addressed then are:

- how far the economy approximates to a closed, atomic system, allowing formal methods, and
- how to theorise about those aspects of the economic system which cannot be so approximated.

For formalist pure theorists, the first is not an issue, and thus neither is the second. The driving force is mathematisation. There may even be a frank admission of lack of correspondence with the real world (see, for example, Hahn, 1973) [10]. But for formalist applied theorists (and in the extreme the application to market activity, as in the Black-Scholes case), the issues are real. Attempts are continually made to increase the realism of the models, within what is possible formally. In practice, too, the formal analysis of 'official discourse' is supplemented by informal methods in 'unofficial discourse' which remains unacknowledged because it goes against the espoused principles of formalism. The limits to formalism are made much more explicit by non-formalist economists, and the application of non-mathematical-formalist methods is justified in terms of the nature of the subject matter.

Critics of mathematical formalism open up the possibility of different types of mathematics. Since the argument really starts with logic, we turn now to consider the issue in terms of different approaches to logic.

3.2. Mathematics and logic

Keynes's first work (Keynes, 1973) [13] addressed the problem of induction, in reaction against Russell and Whitehead's attempts to construct mathematical logic on rationalist grounds. He was concerned with how we establish reasonable grounds for belief in the absence of the conditions for certainty. Certainty for Keynes was the special case, only possible within a closed, atomic structure (in mathematics or in reality), ie those to which classical logic apply. Use of mathematics (based on classical logic) therefore requires justification in terms of the degree to which the case approximates to a closed, atomic structure. Keynes supported the use of mathematics as an aid to thought but argued that the onus was on the economist to demonstrate that its application was appropriate to the subject matter.

Axiomatisation is a type of formalism which relies particularly on classical logic, and which characterises the formalist approach to pure economic theory. Correspondence between theory and reality occurs only at the level of the axioms and at the level of the propositions which emerge from the application of deductive logic. There has been much discussion of the realism or otherwise of the axioms (see for example Hausman, 1992) [12]; the issue of testing we will address in the next subsection.

But there is in addition the question of the logical structure which lies between axioms and testing. Keynes argued that, in the face of uncertainty, we employ what he called ordinary logic (or human logic). This logic, unlike classical logic, is non-demonstrable. It involves building up evidence and constructing indirect knowledge (for a scientist, we would call this knowledge theoretical) as far as possible. But, since this would in general be insufficient as a basis for action, we supplement this knowledge with the aid of convention and also intuition, or imagination.

What this implies for the methodology of economics is reliance on a variety of methods, some of which will be non-mathematical. Only if all methods are mathematical in the classical-logic sense would the methods be commensurate, ie they could be put together to form a single mathematical system. The methods thus are not in general commensurate and judgement must be employed in order to form a basis for action. Keynes is quite explicit about what he sees as the danger of mathematics in economics:

> "*It is a great fault of symbolic pseudo-mathematical methods of formalising a system of economic analysis . . . that they expressly assume strict independence between the factors involved . . . ; whereas, in ordinary discourse . . . we can keep "at the back of our heads" the necessary reserves and qualifications . . . in a way in which we cannot keep complicated partial differentials "at the back" of several pages of algebra which assume that they all vanish*" (Keynes, 1998).

There remain possibilities for exploring the use of mathematics not based on classical logic. Thus, for example, fuzzy mathematics would appear to address some of the concerns of open-system theorising. The mathematics of chaos theory attracted attention for some time because it allowed a formal analysis of disequilibrium behaviour. But it has proved to be unsatisfactory in offering only the chaotic dual of stability. More promising would be the mathematics for analysing the self-organising systems of chemistry which better captures the capacity of social systems to adapt to episodes of particular instability. There may be other developments in mathematics outside the mould of classical logic which could assist the social sciences. But, since the requirement is for a mathematics which can handle open systems, it is inevitable that it will not provide the complete answer to economic methodology; if our subject matter evolves and is creative and purposeful, there is inevitable uncertainty on the part of the economist as much as economic agents. Thus, while Anderson (1988) put forward mathematical techniques for dealing with complex systems, Chick (1998) points out that:

> "*In a complex system, results obtained through a narrow focus do not have general validity . . . . The more complex the system, the greater our ignorance of all the interactions taking place. Neither the actions of agents within the system nor the study of the system from outside can be fully informed. Perfect knowledge is not available.*"

Much of the discussion of mathematics in economics focuses on the testing of economic propositions, i.e. on the design and use of econometrics. We consider the particular issues raised by econometrics in the next subsection.

3.3. Mathematics and econometrics

Econometrics is the name given to the set of statistical techniques employed to test economic theories, or, increasingly, as a means of presenting 'the facts'.

We have noted the difficulties faced by pure theorists in identifying empirical counterparts to theoretical concepts, such as utility. A variety of developments offered solutions to this problem, by avoiding it. Thus, while it was impossible to test directly the detailed mathematical reasoning behind the negative relation between price and quantity demanded, evidence of such a relation was seen as adequate justification for the underlying reasoning, however unquantifiable (Samuelson's Revealed Preference theory). Similarly, while the testing of macro-economic relationships required the existence and uniqueness of equilibrium, something which could not be tested directly, the observed relative stability of economies was taken as justification (Samuelson's Correspondence Principle).

Some econometric models have been extremely elaborate, involving systems of hundreds of equations representing relationships in different sector of the economy. They are still restricted to those variables which are identifiable and measurable. These large models are now out of fashion, given their poor record in prediction, and the norm is more narrowly-defined models. The testing takes the form of estimating relationships which are more or less reduced-form. The assumption is that the posited structure of relationships is stable over the estimation period and that the data are drawn from a probability distribution.

Keynes had argued that the econometrician must justify application of econometric techniques with reference to the subject matter. He argued that, since economic structure in general evolves, econometric techniques cannot in general be applied. His logic was based on the argument that the general case for social systems in particular is not one for which probabilities can be quantified, so that a more general concept of probability is required.

In practice, econometrics is not entirely formal, although there have been attempts to formalise the process. In selecting data, and in the formulation of relationships to be tested, economists employing econometrics bring to bear a whole variety of additional considerations. Nevertheless tremendous store is put by the formal outcome of the econometric process, and the informal input suppressed.

The purpose of much of applied economics is to provide a basis for policy advice. To the extent that mathematical formulation inevitably rules out considerations which cannot be expressed mathematically, the policy application of mathematical models poses important questions. All theory must abstract, but is the type of abstraction required by mathematical expression particularly significant? This is a large, controversial question on which much of the previous discussion bears. Much depends on whether the outcome of the mathematical model and its empirical estimation are regarded as definitive or only partial, to be considered along with other types of knowledge arrived at using different methods.

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